Modelling 1

SUMMER TERM 2020

Institut für Informatik

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

LECTURE 15

Robust Fitting & IRLS

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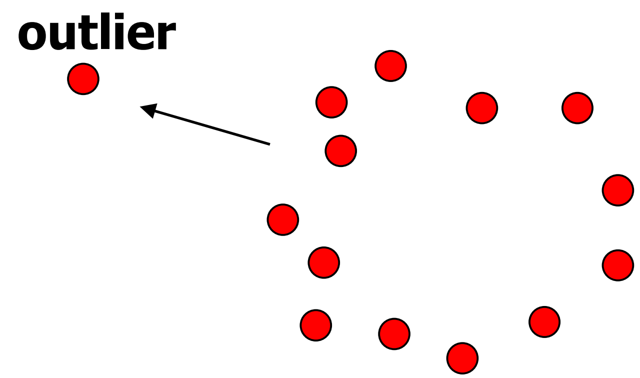
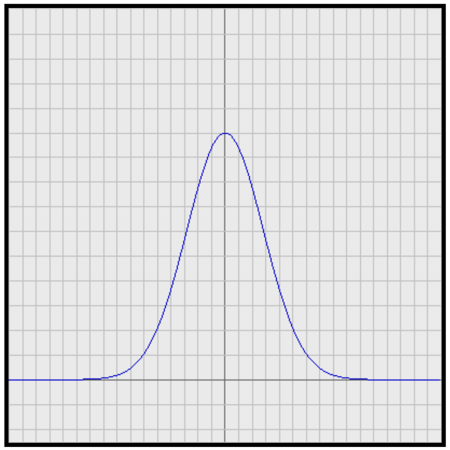
Iteratively Reweighted Least Squares

General Error Distributions

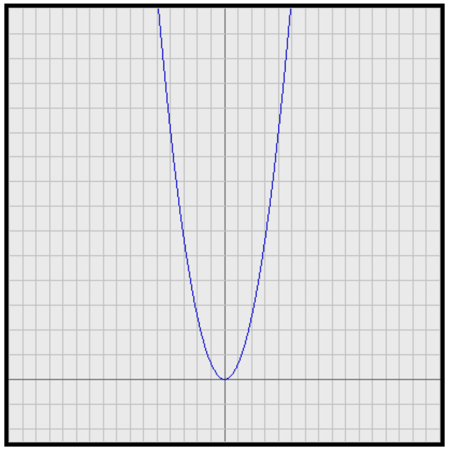
Problem with least-squares

- Quadratic error measure
- Farther away → stronger force
- Outliers are disastrous

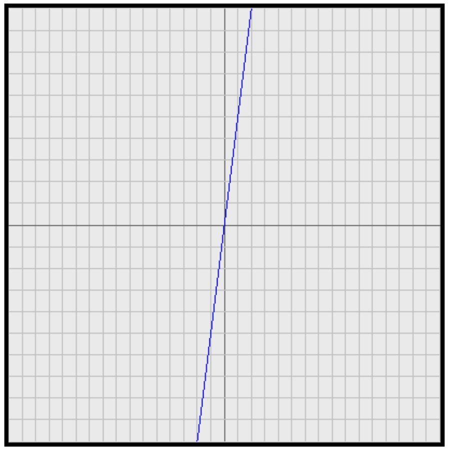




Gaussian likelihood



neg. log-likelihood

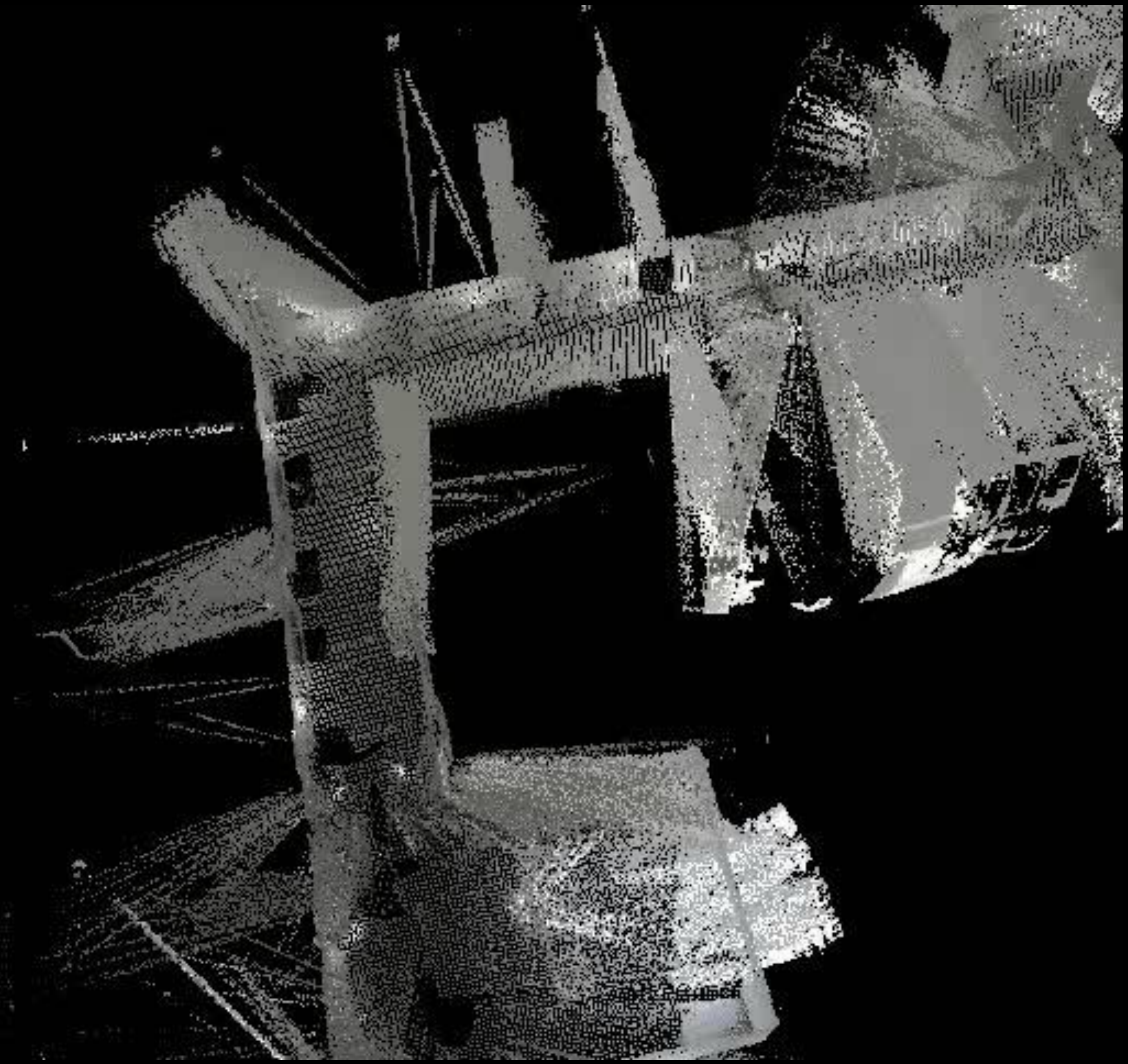


derivative

Automatic Outlier Removal

Data set: Sven Fleck, Peter Biber, Tuebingen University

Example in Practice



General Error Distributions

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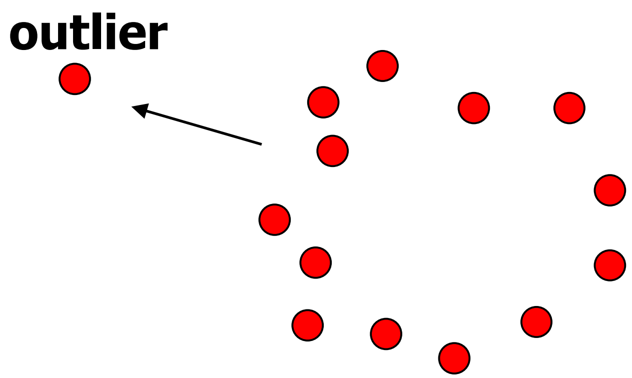

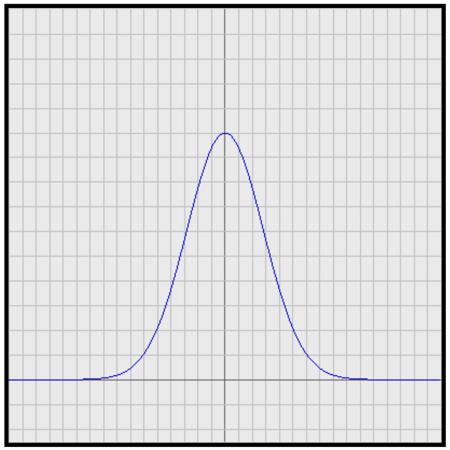

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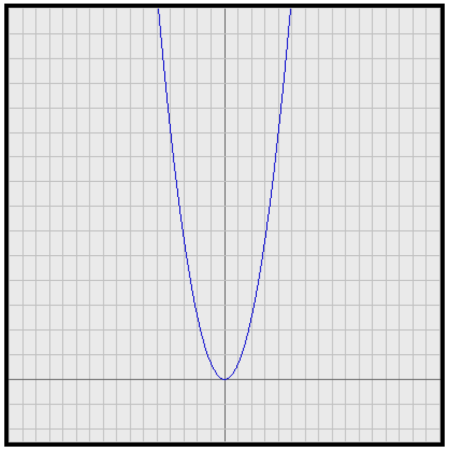

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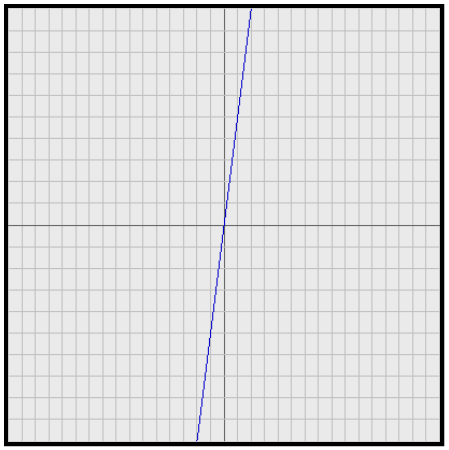
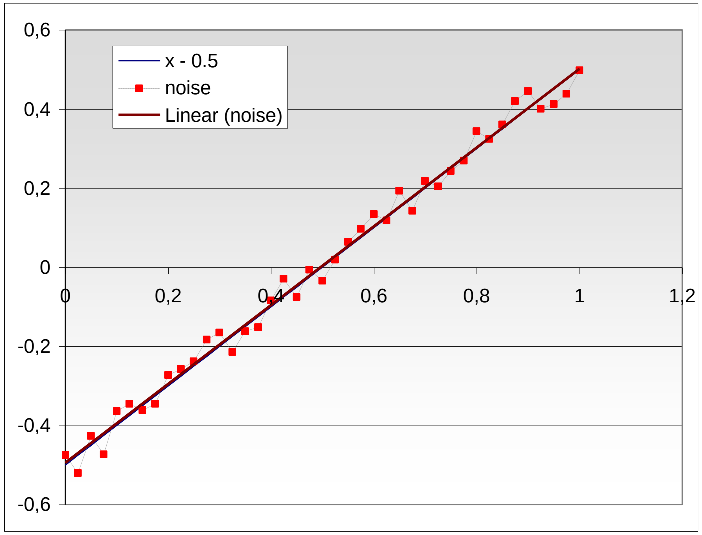
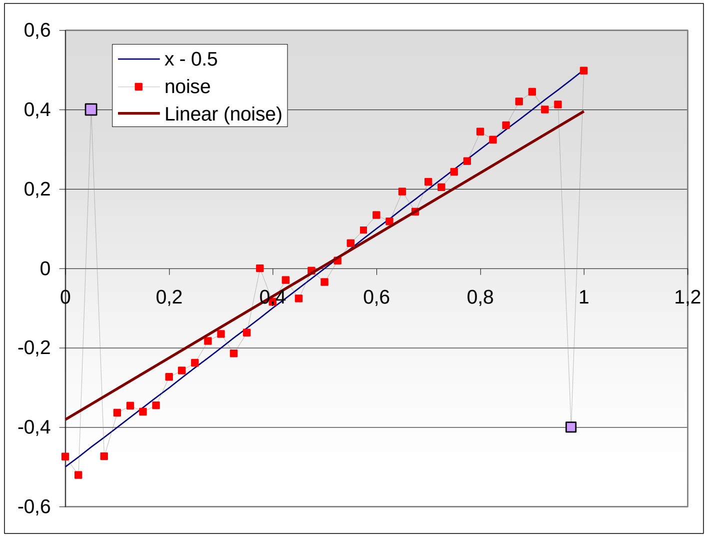


derivative

Outliers



uniform noise

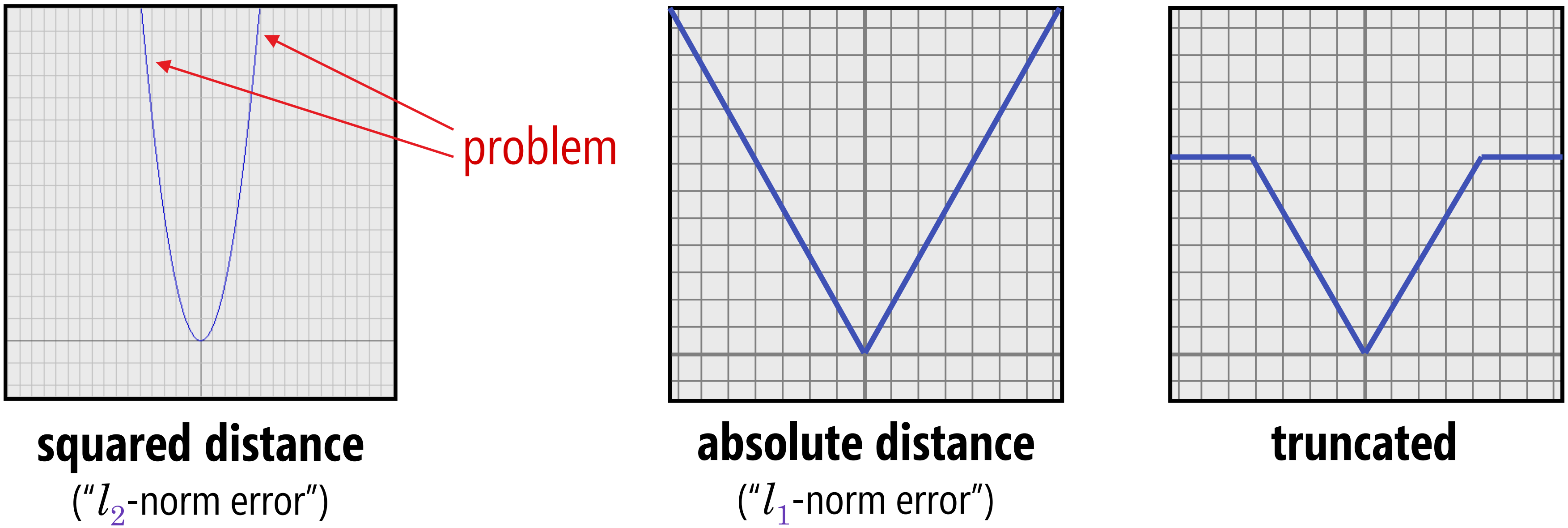


2 outliers (out of 41)

Problem:

- Outliers are rather common
  - 3D scanners: Weird outlier points at random locations (Shiny surfaces, transmission errors, etc...)
- Least squares does not work well

Robust Estimators



Problem:

- Least squares criterion too strict
- More robust: absolute distance ("l_1-norm")
- "M-estimator": Truncated squared/absolute distance

Iteratively Reweighted Least Squares

Simple Implementation

Simple Extension: Iteratively reweighted least-squares

- Reminder: weighted least-squares

$$\arg\min_{\tilde{f}} \sum_{i=1}^{n} \underbrace{\omega_i}_{\text{weights}} \left(\tilde{f}(x_i) - y_i\right)^2$$

- Iteratively reweighted least-squares
  - Compute least-squares fit
  - Compute weights (dependent on solution)
  - Recompute / iterate until convergence
- L1 norm (absolute distance) weights

$$\omega_i = \frac{1}{\left\|\tilde{f}^{(k-1)}(x_i) - y_i\right\|} \rightarrow \arg\min_{\tilde{f}^{(k)}} \sum_{i=1}^{n} \underbrace{\omega_i}_{\text{weights}} \left(\tilde{f}^{(k)}(x_i) - y_i\right)^2$$

(iteration k)

L1-Norm Fitting

- Iterating $\arg\min_{\tilde{f}^{(k)}} \sum_{i=1}^{n} \frac{1}{\left\|\tilde{f}^{(k-1)}(x_i) - y_i\right\|} \left(\tilde{f}^{(k)}(x_i) - y_i\right)^2$
- Convergence guaranteed
  - Global minimum
  - (convex objective function)
- Mixture of l_1 (far) and l_2 (near) norm also works
  - Any norm l_p, $p \geq 1$ is convex

$$\sum_{i=1}^{n} \left|\tilde{f}(x_i) - y_i\right|^p \quad \text{with} \quad p \geq 1$$

M-Estimators

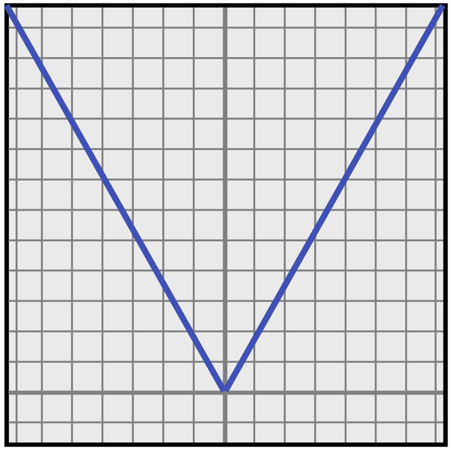
General M-Estimators

- $p < 1$ even more robust
- Huber estimator
  - Piecewise mixtures of l_1, l_2 (often with truncation)
    - Least-squares for small errors,
    - l_1 for medium devations
    - Extension: truncation for large outliers

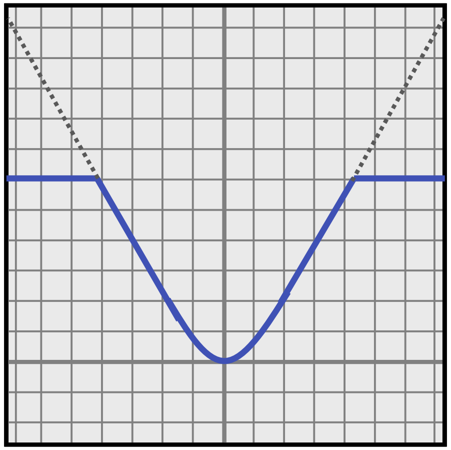
Non-convex potentials

- Convergence to global minimum not guaranteed
- Local extremum; depends on initialization

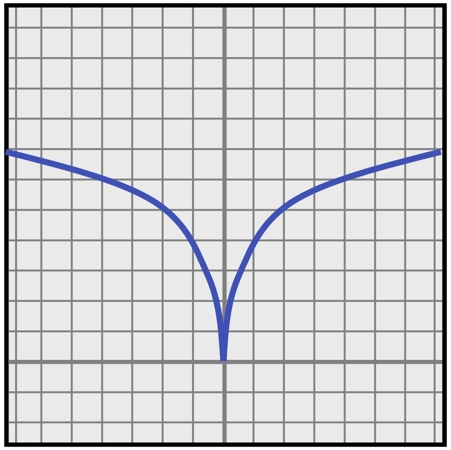
Robust Estimators



absolute distance
("l_1-norm error")

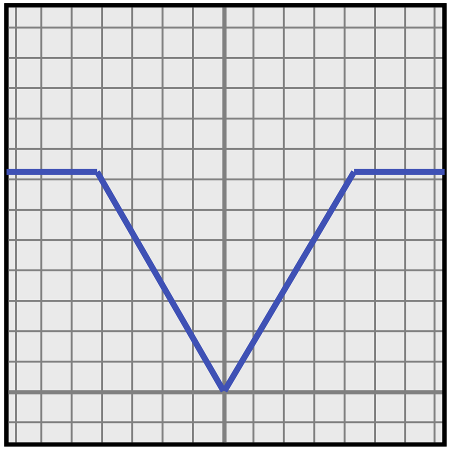


Huber
(mixed l_2, l_1, truncated)



Square-Root
($||\Delta y||^{0.5}$)

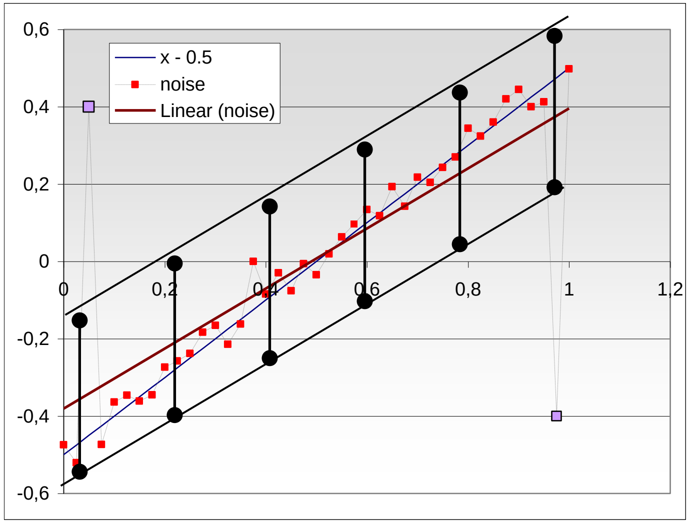
convex



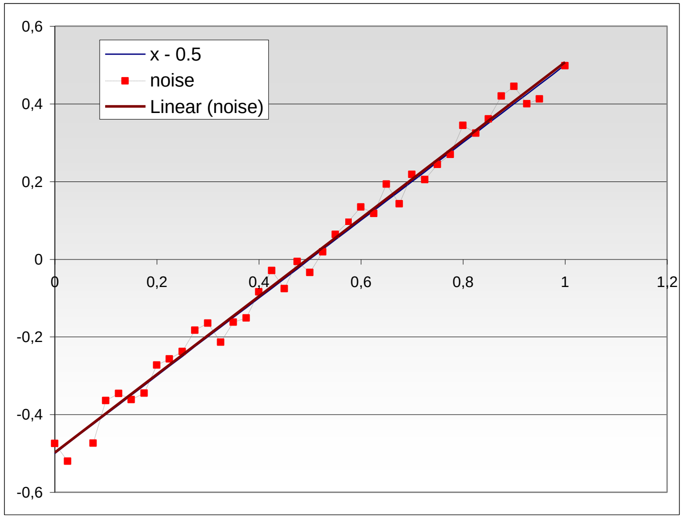
l_1 truncated

non-convex

Example (Schematic)



first iteration:
least squares, then truncation



second iteration:
improved solution